1) $\frac{1}{4} - \frac{1}{6} + \frac{1}{8} - \frac{1}{10} + - \dots = \sum_{n=2}^{\infty} (-1)^n / 2n$

Note $1/2n$ is decreasing and $\lim_{n\to\infty} 1/2n = 0$, so by the alternating series test, the series converges.

2) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt[4]{n}}$ $\qquad \cos(n\pi) = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases} = (-1)^n$

$= \sum_{n=1}^{\infty} (-1)^n / \sqrt[4]{n}$ $\qquad$ Since $\lim_{n\to\infty} \frac{1}{\sqrt[4]{n}} = \lim_{n\to\infty} n^{-1/4} = 0$

and $n^{-1/4} \geq (n+1)^{-1/4}$ (the sequence is decreasing),

the series converges by the alternating series test.

3) $\sum_{n=1}^{\infty} (-1)^n n^3 / 5+n^4$ $\qquad$ Since $\lim_{n\to\infty} n^3/5+n^4 = \lim_{n\to\infty} \frac{1}{n+5/n^3} = 0$

the series converges by the alternating series test

and $\frac{1}{n+5/n^3} \geq \frac{1}{n+1+\frac{5}{(n+1)^3}}$ for $n>1$

as $n+1+\frac{5}{(n+1)^3} \geq n + \frac{5}{n^3}$

$\iff 1 + \frac{5}{(n+1)^3} \geq \frac{5}{n^3}$

$\iff n^3 + \frac{5n^3}{(n+1)^3} - 5 \geq 0$ for $n>1$

4) $\sum_{n=2}^{\infty} (-1)^n / n\ln(n)$ $\qquad$ Since $\lim_{n\to\infty} \frac{1}{n\ln(n)} = 0$

the series converges by the alternating series test

and $\frac{1}{n\ln(n)} \geq \frac{1}{(n+1)\ln(n+1)}$

as $n < n+1$, $\ln(n) < \ln(n+1)$

5) $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n-1}$ $\qquad$ Since $\lim_{n\to\infty} \frac{3n-1}{2n-1} = \frac{3}{2}$,

the series diverges by the divergence test

we see $\lim_{n\to\infty} (-1)^n \frac{3n-1}{2n-1}$ diverges.

6) $\sum_{n=1}^{\infty} (-1)^n \left(1-\cos\left(\frac{\pi}{n}\right)\right)$

the series converges by the alternating series test.

Since $\lim_{n\to\infty} \left(1-\cos\left(\frac{\pi}{n}\right)\right)$

$= 1-\cos\left(\lim_{n\to\infty} \frac{\pi}{n}\right) = 1-\cos(0) = 0$

$1-\cos\left(\frac{\pi}{n}\right) \geq 1-\cos\left(\frac{\pi}{n+1}\right)$

as $\cos\left(\frac{\pi}{n+1}\right) > \cos\left(\frac{\pi}{n}\right)$

and $\frac{d}{dx}\cos(x) = \sin(x) > 0$

for $0 < x < \pi$,

$\frac{\pi}{n+1} < \frac{\pi}{n}$

7) $\sum_{n=1}^{\infty} \left(\frac{-5}{n}\right)^n = \sum_{n=1}^{\infty} (-1)^n \left(\frac{5}{n}\right)^n$

The series converges by the alternating series test.

Since $\lim_{n\to\infty} \left(\frac{5}{n}\right)^n = 0$

and $\left(\frac{5}{n}\right)^n \geq \left(\frac{5}{n+1}\right)^n > \left(\frac{5}{n+1}\right)^{n+1}$

for $n \geq 5$.

Alternatively,

$\lim_{n\to\infty} \left|\left(-\frac{5}{n}\right)^n\right|^{1/n} = \lim_{n\to\infty} \left(\frac{5}{n}\right)^{n/n} = \lim_{n\to\infty} 5/n = 0,$

so the series converges by the root test. Note the use of absolute values.

8) What is the value of K needed to guarantee $\sum_{n=1}^{K} \frac{(-1)^n}{n^2}$ is within $\frac{1}{1000}$ of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$?

Recall $\left| \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} - \sum_{n=1}^{K} \frac{(-1)^n}{n^2} \right| \leq \frac{1}{(K+1)^2}$,

so we want $\frac{1}{(K+1)^2} \leq \frac{1}{1000}$, or $1000 \leq (K+1)^2$.

Then, $\sqrt{1000} - 1 \leq K$. Since $\sqrt{1000} = 10\sqrt{10} \approx 31.6$,

we see $30.6 \leq K$, so we choose $K=31$.